UDC 621.391

NOISE IMMUNITY CODES DISPLAY IN RELIABILITY CORDINATES

Leonid O. Uryvsky, Anton M. Pieshkin
National technical university of Ukraine "KPI", Kyiv, Ukraine

Background. Noise immunity codes are widely used in the modern telecommunication systems. The methodology of properties count of convolutional codes, offered by authors before, to the equivalent parameters of block codes gave the opportunity to use the known and new methodologies of estimation of noise immunity block codes for the corresponding estimations of convolutional codes.

Objective. The purpose of this paper is the estimation of maximum possibilities of block and convolutional noise immunity codes in the reliability co-ordinates on the basis of accordance of their equivalent parameters.

Methods. Based on Plotkin's and Varshamov-Gilbert's boundaries usage the lines of theoretic error-correcting boundary of block and convolutional codes are built in reliability coordinates. The lines allow finding maximum channel probability of error for which error-correcting code exists that can ensure necessary reliability.

Results. Due to search methodic of convolutional code parameters that are equivalent by correcting abilities to block code parameters, complex assessment of error-correcting, informational and forming complexity properties is made on different kinds of error-correcting codes.

Conclusions. For each probability of symbol errors in a communication channel it is possible to estimate the expedience of the use of block and convolutional codes from the standpoint of the necessary reliability provision, informative efficiency and complication of encoding. Investigational limits allow defining maximally possible probability of error, at which there is a code, able to provide necessary authenticity on the output of decoder.

Key words: noise immunity codes; block and convolutional codes; maximum possibilities of error-correcting; Plotkin's and Varshamov-Gilbert's boundaries; error of symbols; necessary reliability.

Introduction

Error-correcting codes that allow information transfer through the channel with certain reliability are widely used in modern telecommunication systems.

All plenty of error-correcting code subsets of block and convolutional codes may be distinguished. The most widespread convolutional codes are the codes where Viterbi decoding algorithm is utilized. They are used in such wireless technologies as IEEE 802.11, IEEE 802.16, far Space Communications CCSDS, satellite communications TIA-1008 and others.

Among block codes the most widely spread are Reed–Solomon and LDPC codes, which are used in such technologies as DVB-H/T/S, DVB-S2, 802.11n, 802.16e.

According to Shannon's theorem, if source productivity is less then channel throughput, there is a code that can ensure necessary reliability. The theorem certainly holds for $n \to \infty$. For finite n there is a question relating to limits: which input probability of error $\boldsymbol{p}_{\text{in}}$ of discrete sequence can be transformed to necessary reliability $\boldsymbol{p}_{\text{b}}$ (bit-error) at the decoder output? It is naturally to assume that limited block length imposes restrictions on the difference $\boldsymbol{p}_{\text{in}}$ and $\boldsymbol{p}_{\text{b}} < \boldsymbol{p}_{\text{in}}$. Therefore error-correcting capabilities of block codes are limited.

Until today potential capabilities of block codes were measured by known Plotkin boundary and additional to it Varshamov-Gilbert boundary in the Hemming distance coordinates, but not in reliability coordinates of discrete sequence at the decoder output [1-3].

There is no methodic to measure bounds of convolutional codes.

At [4,5] methodic of convolutional code parameters recalculation to equivalent block code parameters is proposed. It allows us to use already known as well as new techniques of block codes assessment for corresponding assessments of convolutional codes.

The purpose of this research is the assessment of boundary capabilities of block and convolutional error-correcting codes in reliability coordinates based on corresponding parameters of equivalency.

To achieve the goal the problem of convolutional and block code parameters comparison is solved. As the next step the methodic of immunity codes display in reliability coordinates is designed.

Problem statement

Let's define main parameters of convolutional and block codes

Main parameters of block codes are:

- n – block length;
- k – the number of informational bits;
- $r = n - k$ – the number of redundant bits;
- $t\ (n)$ – correcting ability of the code – the number of bits, that may be corrected from code block n;

- $d \geq 2t + 1$ – code distance;
- $R = \frac{k}{n}$ – code rate.

Main parameters of convolutional codes are:

- D – the code length restriction, which corresponds to the number of bits in the shift register encoder and describes the complexity of coding;
- R – code rate.

From the parameters below it is obviously that the only common parameter of convolutional and block codes is the coding rate R, which characterizes the informational effectiveness.

Thus, the intermediate task will be display of convolutional and block codes in common coordinates.

Suppose that discrete communication channel is given, which is determined by the probability of error $\boldsymbol{p}_{\text{in}}$. There is also a requirement to necessary reliability, which is characterized by probability of error at the decoder output. In case considered as an example, $\boldsymbol{p}_{\text{b}} \leq 10^{-6}$.

Using known methodic of synthesis of error-correcting codes [4,5] equivalent parameters of convolutional codes in the space of block codes should be determined.

Based on the technique of block codes display in reliability coordinates the results are applied to the case of convolutional codes. The problem of particular interest is search of probability of error boundary. According to the Plotkin boundary there is an area where no codes exist which are able to satisfy necessary reliability.

Definition of equivalent parameters of convolutional codes displayed in the space of block codes

To solve the stated problem error-correcting lines of convolutional codes should be constructed as the first step.

By using hard bit decision Viterbi algorithm the lines are upper boundaries which are found based on weight comparison of transmitted sequence and possible sequences at the decoder input in case of error in the communication channel.

Upper probability boundary at the decoder output is defined from considerations that occurrence probability of one, two or more errors in the channel is less than sum of probabilities of events of occurrence one, two or more errors:

$$\boldsymbol{p}_{\text{b}} \leq \sum_{k=d_f}^{\infty} W_k \cdot P_k, \tag{1}$$

where W_k – coefficients of code spectrum, that are equal to number of errors at the decoder output which occur when received code sequence differ from transmitted sequence by distance $k = d_f$;

P_k – choice probability of wrong path of k weight, is defined by the formula:

$$P_k = \begin{cases} \sum_{i=\frac{k+1}{2}}^{k} C_k^i \cdot p^i \cdot (1-p)^{k-i} - \ for\ odd\ k \\ \frac{1}{2} \cdot C_k^{\frac{k}{2}} \cdot p^{\frac{k}{2}} (1-p)^{\frac{k}{2}} + \sum_{i=\frac{k}{2}+1}^{k} C_k^i \cdot p^i \cdot (1-p)^{k-i} - \ for\ even\ k \end{cases}, \tag{2}$$

where $p = \boldsymbol{p}_{\text{in}}$ – channel probability of error;

$C_k^i = \frac{k!}{i!(k-i)!}$ – number of combinations from k to i.

Let's plot error-correcting lines of convolutional code – function of probability of error at the decoder output from channel (input) probability of error

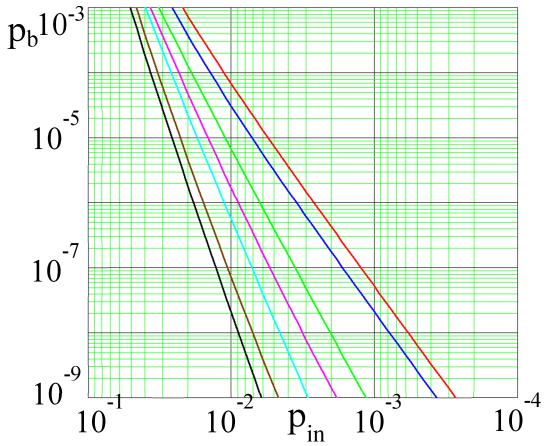

Fig.1. Error-correcting lines for convolutional codes as dependency of probability of error $\boldsymbol{p}_b$ at the decoder output from channel (input) probability of error $\boldsymbol{p}_{in}$ for code rate $R = \frac{1}{2}$, and code length limit $D \in [2,\ 3 \ldots 8]$ right to left

The second step is to define the channel probabilities of error for given requirements to reliability for each convolutional code rate $R \in \{\frac{1}{3}; \frac{1}{2}; \frac{2}{3}; \frac{3}{4}\}$. An example of channel probabilities definition is shown on the Fig. 2

Thus, the second step result is the sets of code rate, channel probability and code length restriction [R; $\boldsymbol{p}_{\text{in}}$; D].

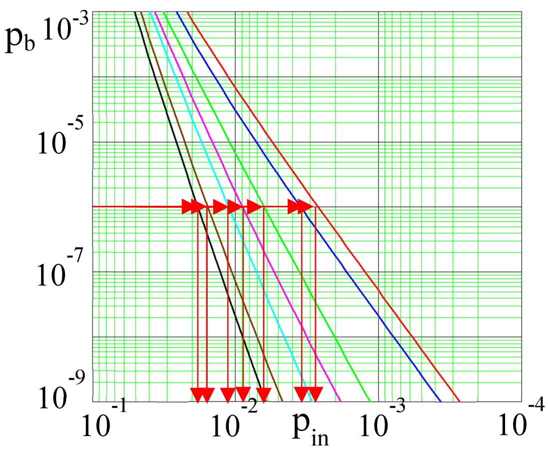

Fig.2. Display of search technique of channel (input) probabilities of error $\boldsymbol{p}_{in}$ of convolutional codes for given input reliability $\boldsymbol{p}_{\text{b}} = 10^{-6}$

The third step is the construction of found parameters of convolutional codes in the coordinates (p_{in}, R). The coordinates are common for convolutional and block codes thus block code and Shannon boundaries can be plotted in the same coordinates for specified n.

The fourth step is the construction of convolutional code projection in the area of error-correcting line of block codes.

The last step sets the parameters compliance between each convolutional code [*R*; *D*] and block code [*n*; *k*; *d*].

Block codes display in reliability coordinates

Let's plot dependency of channel probability of error from block length for given requirements to reliability (fixed probability of error at the decoder output $P = 10^{-6}$) to estimate boundary corrective abilities of block codes.

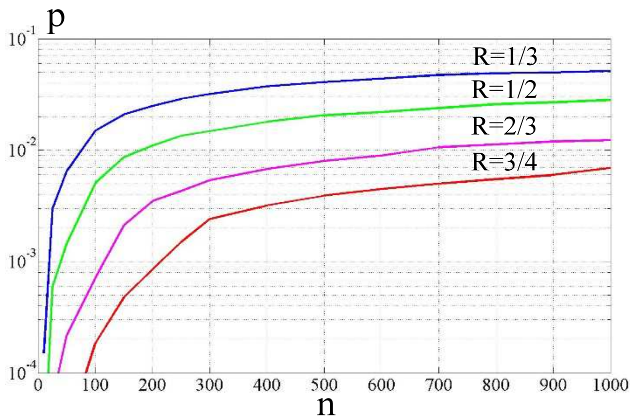

Fig.3. Display of block codes parameters in reliability coordinates $\boldsymbol{p}(\boldsymbol{n})|\boldsymbol{R} \in \{\frac{1}{3};\frac{1}{2};\frac{2}{3};\frac{3}{4}\}$

It is proposed to use code length n as the key parameter of error-correcting codes. For example, to achieve reliability $\boldsymbol{p}_{\mathrm{b}} = 10^{-6}$ in the channel with probability of error $p_{in} = 10^{-3}$ block code can be used with rate $R = \frac{1}{3}$ and length $n = 15$, or $R = \frac{1}{2}$ and length $n = 60$, or $R = \frac{2}{3}$ and length $n = 115$, or $R = \frac{3}{4}$ and length $n = 210$, or the convolutional code can be used with rate $R = \frac{3}{4}$ and code length restriction $D = 8$.

Examples of different error-correcting codes utilization in the channel with common probability of error and requirements to reliability are shown in the table 1.

Table 1. Lengths of codes with different rates that are able to satisfy common probability of error $\boldsymbol{p}_{\mathrm{in}}$ ($\boldsymbol{p}_b = 10^{-6}$)

p_{in}	*1/3*	*1/2*	*2/3*	*3/4*
0,001	15	60	115	210
0,005	50	120	370	800
0,02	120	460	x	x
0.05	1000	x	x	x

According to table 1, the same probability of error may be fixed by the codes of different rates to satisfy necessary reliability. The price for better rate (that is informational effectiveness) is higher complexity (that is coding speed).

Also it should be mentioned that there are boundary probabilities of error, when it is impossible to use codes of high rates (for example $R = \frac{3}{4}$) and satisfy necessary reliability at the same time. In this case code of lower rate have to be chosen (for example $R = \frac{1}{2}$). This statement illustrated in Table 1. The cases where no error-correcting code exists are marked by 'x'. Then code length restriction *n*=1000 can be used.

Exceptional interest is the task to search the boundary of the decoder output error probability, for that no error-correcting code exists, which can satisfy given reliability.

The lowest code length restriction n may be obtained by search of the noise immune code which has lowest code rate $R = 0$. This case is depicted on the Fig.4.

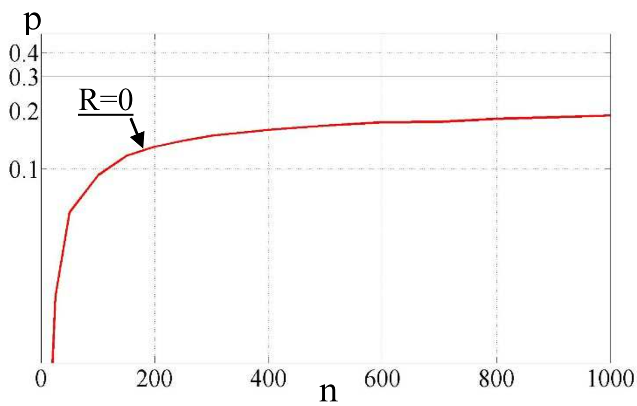

Fig.4. Theoretical existence boundary of error-correcting codes

Dependency on the FigFig.4 corresponds with the intersection point of Plotkin and Varshamov-Gilbert boundaries

Error-correcting codes that belong to the depicted line have the highest error immunity but the code rate $R = 0$. Using the dependency it can be stated that there are no convolutional codes or block codes with block length $n < 1000$ which can ensure probability of error at the level of $\boldsymbol{p}_{\mathrm{b}} = 10^{-6}$ if channel probability of error $\boldsymbol{p}_{\mathrm{in}} \geq 0{,}18$.

For any code length restriction n, using the technique below and depicted line Fig.4 it is possible to find boundary of achievable reliability $\boldsymbol{p}_{\mathrm{b}}$ for the worst channel probability of error $\boldsymbol{p}_{\mathrm{in}}$ when existence of error-correcting code is guaranteed. There are no error-correcting codes outside the boundaries.

In the reliability coordinates the boundary capabilities of error-correcting codes are obtained for the first time.

Convolutional codes display in reliability coordinates

Using block codes display technique in reliability coordinates it is possible to obtain the same result for the case of convolutional codes. In this case equivalent parameters of convolutional codes obtained by Varshamov-Gilbert boundary were used.

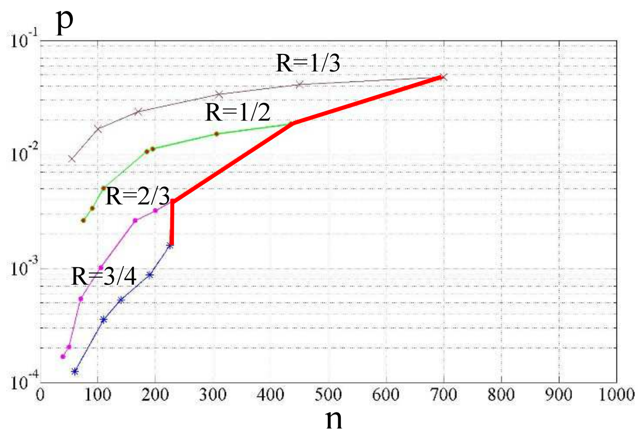

Fig.5. Display of convolutional code parameters in reliability coordinates $\boldsymbol{p}(\boldsymbol{n})|\boldsymbol{R} \in \{\frac{1}{3};\frac{1}{2};\frac{2}{3};\frac{3}{4}\}$

Fig.5 shows probability boundary for convolutional codes within which (left) it is possible to improve given channel probability of error to the level of $\boldsymbol{p}_{\mathrm{b}} = 10^{-6}$ at the decoder output. This boundary is plotted through convolutional codes with the highest D, which have the greatest noise immunity. To fix errors with higher probability block codes should be used only, which have higher block length.

Conclusions

In the research based on known Plotkin and Varshamov-Gilbert boundaries lines of theoretical boundary of error-correcting codes are displayed in reliability coordinates.

Using obtained result for each channel probability of error it is possible to assess appropriateness of block or convolutional code usage to achieve given reliability, informational effectiveness and coding complexity. Also it is possible to choose alternative error-correcting code using specified criteria and restrictions [4,6].

Boundaries allow us to find the maximum probability of error $\boldsymbol{p}_{\mathrm{in}}$ under which there is still code, able to satisfy necessary reliability $\boldsymbol{p}_{\mathrm{b}}$.

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Received in final form on November 17, 2015

Уривський Л. О., Пєшкін А.М.

Відображення параметрів завадостійких кодів в координатах достовірності

Проблематика.В сучасних телекомунікаційних системах широко використовуються завадостійкі коди. Раніше запропонована авторами методика перерахунку властивостей неперервних кодів до еквівалентних параметрів блокових кодів дала змогу використовувати відомі та нові методики оцінки завадостійкості блокових кодів для відповідних оцінок неперервних кодів.

Мета досліджень. Метою даної роботи є оцінка граничних можливостей завадостійкості блокових та неперервних кодів у координатах достовірності на підставі відповідності їх еквівалентних параметрів.

Методика реалізації. На підставі використання відомих границь Плоткіна та Варшамова–Гільберта для корегуючих можливостей блокових кодів в координатах «відстань Гильберта –швидкість кодування» здійснено побудову ліній теоретичної границі корегуючих можливостей блокових та неперервних кодів в координатах достовірності. Досліджені границі дозволяють визначити ту максимально можливу канальну ймовірність помилки, при якій ще існує код, здатний забезпечити необхідну достовірність.

Результати досліджень. Завдяки методики пошуку параметрів неперервних кодів, еквівалентних за коригуючою здатністю відповідним блоковим кодам, виконується комплексна оцінка корегуючих властивостей кодів різних видів разом із оцінкою інформаційної ефективності та складності кодування.

Висновки. Для кожної ймовірності помилок символів у каналі зв’язку можливо оцінити доцільність використання блокового чи неперервного коду з точки зору забезпечення заданої достовірності, інформаційної ефективності та складності кодування. Досліджені границі дозволяють визначити ту максимально можливу ймовірність помилки символів у каналі зв’язку, при якій ще існує код, здатний забезпечити необхідну достовірність на виході декодера.

Ключові слова: завадостійкі коди; блокові та неперервні коди; граничні можливості корегування; границі Плоткіна та Варшамова–Гільберта; помилки символів; необхідна достовірність.

Урывский Л. А., Пешкин А. М.

Отображение параметров помехоустойчивых кодов в координатах достоверности

Проблематика. В современных телекоммуникационных системах широко используются помехоустойчивые коды. Ранее предложенная авторами методика пересчета свойств непрерывных кодов к эквивалентным параметрам блочных кодов дала возможность использовать известные и новые методики оценки помехоустойчивости блочных кодов для соответствующих оценок непрерывных кодов.

Цель исследований. Целью данной работы является оценка предельных возможностей помехоустойчивости блочных и непрерывных кодов в координатах достоверности на основании соответствия их эквивалентных параметров.

Методика реализации. На основании использования известных границ Плоткина и Варшамов-Гильберта для корректирующих возможностей блочных кодов в координатах «расстояние Гильберта - скорость кодирования» осуществлено построение линий теоретической границы корректирующих возможностей блочных и непрерывных кодов в координатах достоверности. Исследованные границы позволяют определить ту максимально возможную вероятность ошибки символов в канале связи, при которой еще существует код, способный обеспечить необходимую достоверность.

Результаты исследований. Благодаря методики поиска параметров непрерывных кодов, эквивалентных за корректирующей способностью соответствующих блочных кодов, выполняется комплексная оценка корректирующих свойств кодов разных видов вместе с оценкой информационной эффективности и сложности кодирования.

Выводы. Для каждой вероятности ошибок символов в канале связи возможно оценить целесообразность использования блочного или непрерывного кода с точки зрения обеспечения заданной достоверности, информационной эффективности и сложности кодирования. Исследованные границы позволяют определить ту максимально возможную вероятность ошибки символов в канале связку, при которой еще существует код, способный обеспечить необходимую достоверность на выходе декодера.

Ключевые слова: помехоустойчивые коды; блочные и непрерывные коды; предельные возможности корректирования ошибок; границы Плоткина и Варшамова–Гильберта; ошибки символов; необходимая достоверность.